THE LISTENER CROSSWORD

Notes for Setters of Mathematical Puzzles

Background

It has been regular recent practice for four 'mathematical' puzzles to be published each year, in the last Saturdays in February, May, August and November. On some occasions such a puzzle has not appeared in that slot because of the need to schedule a word-based puzzle on a relevant date.

The most salient feature of Listener crosswords is their variety. This is not, as some assume, simply a variety of theme; we also seek a variety of challenge. There are four main categories of solvers.

1. *Mathematics only.* The statistics quoted at the end of this document show, not surprisingly, a correlation between the number of new entrants and the number of submissions. Having an occasional easy mathematical puzzle encourages solvers who would not otherwise submit a solution, more so than for a word-based puzzle. This can only enhance the status of the series. Perhaps some of these newcomers will persevere with harder puzzles or even attempt word-based ones.
2. *Mathematics enthusiasts.* These are the solvers who tend to provide enthusiastic feedback for difficult puzzles. Their number is relatively small and such feedback gives a false impression of the overall perception of a puzzle.
3. *Mathematics haters.* An unvarying diet of intractable puzzles will merely harden their attitude.
4. *Compulsive solvers.* These will attempt every Listener puzzle, no matter how demanding. Many do derive satisfaction proportional to the challenge they have encountered, but there are a significant number for whom this does not hold for mathematical puzzles. For these, the quality and elegance of the puzzle is more important.

Thus, the editors wish to receive a *variety* of submissions in every sense of the word. They wish to schedule puzzles to reflect that variety and *high quality* puzzles will be given priority. The factors by which that judgement will be made include enjoyment, elegance and uniformity of challenge throughout the solving process, but not the degree of difficulty.

Key features

We regard the following as key features for mathematical puzzles, in current circumstances.

- Solution should be possible with (at most) a simple scientific calculator.
- If solving can be greatly accelerated by using a computer program, that is a bad sign.
- Feedback gained from the 'crossing' aspect of the crossword diagram should be integral to the solution of the puzzle.
- Mathematical puzzles shouldn't be *too* mathematical: they should stick to simple GCSE (or the old 'O' level) Mathematics concepts and achieve originality by using these in innovative ways.
- Word-based puzzles with a non-trivial mathematical element are liable to be classed as mathematical.
- Above all, mathematical puzzles have to be *entertaining* for regular solvers of the word-based puzzles.
- The solving task should be extensive, with logical deduction required until near the end.
- Care must be taken with puzzles where numbers are rounded.
- There must be a fairly obvious starting point.

- Solvers should not be expected to make inspirational deductions about the theme, without adequate hints.

The subsections that follow develop these observations, *in general terms*. Just as for conventional crosswords, 'rules' may be weakened for puzzles that are strikingly original in some way.

Calculators

There is no need to assume that solvers must be able to perform the necessary calculations by hand. It is expected that, for most puzzles, a simple 4-function calculator would suffice, but we wish to allow a scientific one for two reasons. One is the ability to calculate powers and, more importantly, extract roots, eg, to search for perfect powers. The other is that their logic is more likely to correspond to standard mathematical conventions, as in the interpretation of ab+cd, as (ab)+(cd).

Computers

It is most unfortunate that some of the devices used in Listener puzzles in pre-computer days are now potentially trivial. For example, some of these involved finding sets of numbers with certain properties, using a simple starting set plus grid crossings. These can now be solved by finding all possible sets and fitting the grid like a jigsaw, thereby undermining the entire puzzle.

It is, however, impossible to remove all advantages of a computer. Thus, puzzles may still require the identification of prime numbers. The inability of a calculator to match a computer's ability to find quotients and remainders should be regarded as a deficiency of the former rather than an advantage of the latter. Similarly, puzzles may assume an ability to use different number bases, although most calculators cannot do the necessary conversions directly and programs exist enabling computers to do so.

Further, there is no need to worry about situations where a successful program would take as long to write as a normal solution process. For example, a program that took account of grid crossings could be as much an intellectual exercise as the original puzzle. What would be inappropriate is a puzzle based on square triangular numbers, 1, 36, 1225, 41616, 1413721, 48024900, 1631432881, …. These were produced within a few minutes on a computer, but would need hours on a calculator (but see below).

'Crossing' information

Several solvers object to the inclusion of mathematical puzzles in the Listener series on the grounds that they are not cross*words.* This view is not supported by the editors. But they will find it hard to defend a puzzle if it loses the *cross* feature as well. Indeed, it is hard to defend word-based puzzles without such a feature.

Mathematical content

Although the reference to 'O'-level/GCSE is a somewhat an ill-defined notion, it is still a useful 'rule of thumb'. Simple trigonometry would be feasible, but not inverse trigonometric functions. Logarithms ought to be excluded by this rule, but simple cases have made a tentative appearance recently. Certainly simple algebraic manipulation is feasible, as well as knowledge of the properties of whole numbers, including factorisation. Setters should bear in mind that filling gaps in one's mathematical knowledge is much more difficult than tracking down some unknown literary reference.

Many of the old Listener puzzles (before the era of *The Times*) assumed much more mathematical knowledge and hence such puzzles are not always good models. For example, some around 1960 used numbers that were simultaneously square and triangular (see above); the only feasible generation method at the time required University-level number theory.

Word-based content

As noted above, puzzles with both word-based and non-trivial mathematical element are liable to be classed as mathematical; such hybrids tend to be poorly received by solvers and only superlative examples will be published. This includes puzzles where it is necessary to solve word-based clues before switching to the mathematical component. It does not cover puzzles that are principally word-based but involve some sort of enciphering or translation using letter values for words. The statistical data at the end of this document illustrates how few solvers find them of interest.

Entertainment

To say that mathematical puzzles have to be *entertaining* for regular solvers seems unexceptionable, but it does not explain just what makes a puzzle enjoyable and that is likely to vary from solver to solver. A principal requirement for those who solve these puzzles for the buzz obtained by 'defeating' the setter is the matter of the next subsection. Others may give more prominence to elegance, simplicity (of concept rather than execution) and the revelation of intriguing facts.

It is perhaps easier to suggest what prohibits enjoyment. One may think of intractability, a large amount of routine computation to achieve each entry, frequent back-tracking with associated rubbing out and a solution that becomes trivial after a few observations. For some solvers, a suspicion that a mathematician or a computer programmer could avoid their hours of effort, by application of special skills, may sour their appreciation.

One feature that is likely to generate interest is that elusive factor: innovation.

Solution process

The previous section mentioned routine computation as an unpopular feature, but it may be necessary to deliver what dedicated solvers regard as a satisfactory feature. This is the need to solve a succession of sub-puzzles in order to complete the grid. Such puzzles do not get easier as the solution progresses, or at least not until very near the end. The solver thus gains prolonged satisfaction from the process: relishing lots of pennies dropping rather than a single pound coin. It is helpful, however, if the solver gets occasional confirmation of the answers to date.

Rounded numbers

Almost all puzzles have used (positive) whole numbers. A few have ventured into rational numbers (exact fractions). Others have used irrational numbers, which can be represented only by non-terminating decimal expansions. Usually these have been entered rounded and without the decimal point or, equivalently, by using a truncated form of a multiple by a power of 10. Such devices are acceptable provided care is taken in two ways.

Firstly, attention should be paid to the precision of a typical calculator. It is unfair to expect solvers to access, say, the 12^{th} decimal place of sin 40°.

Secondly, arithmetic should be avoided unless it is well-defined. The reason for this is that the familiar rules of arithmetic, such as $(a+b)+c=a+(b+c)$, can fail with rounded computation. Thus, a trigonometric calculation that can be done in two ways may produce a non-unique answer.

Starting point

The vetters will not pursue a puzzle when no obvious starting point can be seen within about 30 minutes. Both vetters were successful solvers of mathematically based puzzles in the Listener series, so if they cannot make a start, it is likely that the puzzle will be too demanding for the vast majority of solvers, who will give up even sooner.

Thematic information

One factor that can lead to difficulties in starting is where a fundamental piece of information has to be gleaned with inadequate hints, eg, letter assignments must be deduced from the title or preamble rather than deduced from clues in the usual way.

If an indirect indication is given it shouldn't be so vague as to be noticeable only when the puzzle has been completed. This is a delicate problem since, of course, setters are aware of such features from the start and cannot look at the finished product with solvers' eyes.

Special requirements for mathematical puzzles

Setters should avoid using letters in clues that could be misconstrued as other letters or numbers: for example, o and O (get confused with 0); x and X (get confused with ×); I and l (get confused with each other and with 1); and so on.

In most cases, multiplication is shown by juxtaposition. If there is a need to make it explicit the use of a period is convenient, A.B, unless there are decimal numbers involved. Superscripts, subscripts and multi-line layout are not possible. Thus, division is best shown by A/B, and powering by A^B. Square roots can be shown by A^(1/2) and cube roots by A^(1/3). In connection with this last point, setters should note that many solvers are not aware of the conventions of fractional and negative indices and others readily confuse them. The factorial function may be used and should be notated as A!, but brackets will be needed for more adventurous use, such as (2B)! or (A+B)!.

The use of such conventions requires care in the order of calculations, remembering that the conventions used by a modern scientific calculator must be followed. Thus, A/BC may appear to give A divided by BC, but is in fact (A/B)×C = (AC)/B; for the alternative version, A/(BC) must be used. Similarly, the square root of A+B must be written using brackets, (A+B)^(1/2), as must its reciprocal, 1/(A+B).

Preference is given to ideas that can be elegantly expressed within the typographical constraints. The editors reserve the right to amend notation to comply with these constraints.

It has been normal, but not essential, that every grid entry is distinct. Feedback suggests that mentioning this in the preamble, eg, adding “Every grid entry is different” when this is the case, would be appreciated by solvers. The stock phrase “No entry starts with zero” should also be stated when it applies.

One final observation: remember that submissions must include a <u>detailed</u> worked solution, which means setting out a route by which a solver could reach a unique answer. Unsuccessful solvers often ask for a model solution, so this document could be used for that purpose, as well as helping the vetters.

The challenge for setters

The principal objectives for setters of mathematical puzzles is that they should stick to simple ‘O’ level or GCSE maths concepts and achieve originality by using these in innovative ways. The best mathematical puzzles are those that are admired at least as much by regular Listener solvers as by those that only attempt mathematical ones.

These objectives sum up what we would be most pleased to find in submissions, but they are much easier to state than to achieve. Furthermore, our experience suggests that setters sometimes overestimate the abilities of the solvers, in spite of the enormous talent Listener solvers share. (This is not a criticism, rather a recognition that once one has spent many hours in constructing a mathematical exercise, it appears much easier than it did at the outset.)

It is hard to be innovative when many of the best ideas have been used already. There may be some mileage in revisiting the best of the older puzzles, adding some new feature. One helpful development has been a recent increase in the number of books devoted to mathematical recreation, although there are also a number of excellent texts that have been on the market for some time. References to some of these are given below.

References [including ISBN]

Wells, Davis: *The Penguin Dictionary of Curious and Interesting Numbers*, Penguin [0140261494]
(This has occasionally been recommended in the series of Listener puzzles.)

Jenkins, Adrian: *The Number File*, Tarquin Press [1899618406]
(Recommended by one of the most prolific current setters.)

Conway, John H. & Guy, Richard K.: *The Book of Numbers*, Springer-Verlag [038797993X]

Beiler, Albert: *Recreations in the Theory of Numbers*, Dover [0486210960]

Gardner, Martin: A large collection of books containing interesting and accessible mathematics.

Stewart, Ian: He has also published several books on this topic.

http://mathworld.wolfram.com : A repository of mathematical topics at a wide range of levels, with many references included.

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Co-editors, The Listener Crossword
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Some statistical information

The ***Number*** of submissions (correct and wrong) for each mathematically-based puzzle in the years 1998–2004 were as below. The column headed ***New*** gives the number of solvers for whom this was the first submission for that year. The ***Average*** includes all puzzles, word and number.

Puzzle Year	**Average Overall Entry**	**Quarter 1**		**Quarter 2**		**Quarter 3**		**Quarter 4**		**Other**	
		Number	New	Number	New	Number	New	Number	New	Number	New
1998	527	452	128	780	317	1243	492	603	77	-	-
1999	451	411	122	-	-	273	70	853	318	-	-
2000	396	488	196	315	64	(156)	(7)	378	75	-	-
2001	515	467	175	1059	479	1050	354	-	-	-	-
2002	538	930	275	562	79	525	70	197	5	754	341
2003	489	453	155	583	156	284	23	597	138	-	-
2004	594	433	114	1035	448	859	214	910	162	-	-

The ***Other*** puzzle was published early in 2002. Otherwise, each puzzle appeared about 2/3 of the way through its quarter. That in Q3 of 2000 was a hybrid, requiring solution of word clues as well as number ones.