The Listener Crossword No 4062

Letter Squares by Elap

It is important to remember that each letter in the clues represents a perfect square.

In this solution the character □ denotes a digit.

One approach to solving this puzzle is as follows.

23ac **E + KK + M (2)**
K can't be greater than 4, because otherwise even if **E** and **M** represented the lowest remaining values possible, 23ac would have three digits. Therefore, **K = 4.**

18ac **K + KK + 2U (2)**
Since **K** is 4, **U** must be 9, 16, 25 or 36, otherwise the value would have more than two digits.

2ac **KK + UU (4)**
Only **U = 36** (the maximum value it could be) is large enough to give a four-digit value, making 2ac = 1312 (entered as 1 3 1 2, 1 3 12, 13 1 2 or 13 12) and 18ac = 92 which can only be entered as 9 2.

18dn **2B (2)**
This begins with 9 and is twice a perfect square, and so **B = 49** and 18dn is 98.

17ac **BU + K + W (4)**
We know that **B** is 49, **U** is 36 and **K** = 4, and so we have 1768 + **W** = □□92. **W** therefore ends with 24. Since the first two digits of the answer must be in a single cell, and therefore in the range 10 to 25 (as stated in the preamble), **W** must be in the range 24 to 824. **W = 324** is the only square which fits, making 17ac = 2092.

19dn **G (4)**
Since 24ac has three digits, and the first digit (8) is already in the grid, **G** must fit into 2 □□ □, and the possibilities are **G** = 2116 or **G** = 2209 (any higher value of **G** would have a value greater than 25 in the second cell).

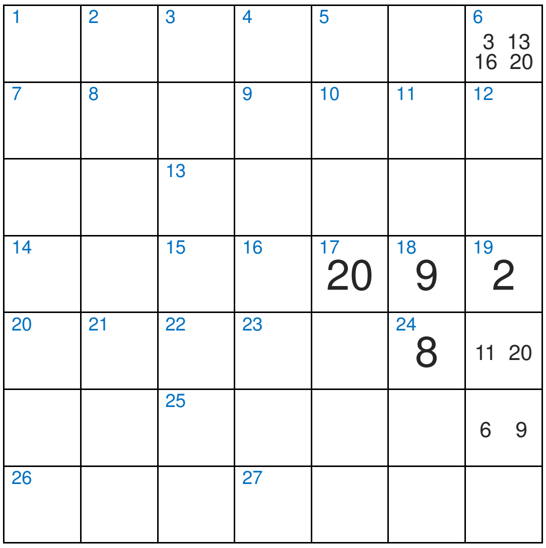
12dn **H + III + T + V (7)**
I (a perfect square) cannot be greater than 196 otherwise 12dn would have more than 7 digits.

5ac **G + 3I + II (5)**
We have **3I + II** + **2116** = □□□□□ or **3I + II** + **2209** = □□□□□ and the result must fit into three cells. **I** cannot be less than 100 (ie, 81 or lower) otherwise 5ac would have less than five digits. The possibilities are

I	3I + II + 2116	3I + II + 2209
100	12416	12509
121	17120	17213
144	23284	23377
169	31184	31277
196	41120	41213

Only **I** = 100, **I** = 121 and **I** = 196 can fit into three cells according to the rules, making 5ac 12416, 17120, 17213, 41120 or 41213. The possibilities for 5ac are therefore 1 24 16, 12 4 16, 17 1 20, 17 2 13, 17 21 3, 4 11 20 or 4 12 13. The third cell of 5ac therefore contains 3, 13, 16 or 20.

This is the current state of the grid:



A =
B = 49
C =
D =
E =
F =
G =
H =
I =
J =
K = 4
L =
M =

N =
O =
P =
R =
S =
T =
U = 36
V =
W = 324
X =
Y =
Z =

6dn **B + S + 2SS (4)**
B is 49, and so we have 49 + **S + 2SS** = 13□□, 16□□, 20□□ or 3□□□. **S** = 9 or **S** = 16 is clearly too small, but **S** = 25 gives 6dn = 1324. **S** can't be 36 because **U** is, and any higher value would be too big. Therefore **S = 25** and 6dn = 1324, to be entered as 13 2 4 or 13 24. Whichever way it is to be entered, 13 can be written in the first cell.

5ac **G + 3I + II (5)** again:
Since the answer ends with 13, **I** = 121 or 196 (not 100) and **G = 2209**, which can be written in the grid at 19dn as 2 20 9 (not 2 2 09, because leading zeroes are not allowed).

12dn **H + III + T + V (7)** again
We know that **I** is 121 or 196. The latter value gives 12dn = 7529536 + **H** + **T** + **V**, but this is too large for 6dn (which is 1324□□□). Therefore, **I = 121**. 5ac is therefore 2209 + 3x121 + 121^2 = 17213, to be entered as 17 2 13. Since 4ac has five digits, its first cell must contain two of them, and so 2ac must be entered as 1 3 12 or 13 12.

14dn **2P + Y + YYY (4)**
Y (a perfect square) can't exceed 16, otherwise 14dn would have more than four digits. **Y** is therefore 9 or 16 (**K** is already 4).

24ac **D + K + 2Y (3)**
24ac is known — it has been provided by 18dn and 19dn — and is 820), and so we have **D** + 2**Y** = 816. **Y = 9** doesn't produce a square value for **D**, but **Y = 16** does, making **D = 784**.

9ac **DY + UU + 2Y (5)**
This can now be evaluated as 13872 and it can only be entered as 13 8 7 2, meaning that 6dn must be entered as 13 2 4 (not 13 24).

23ac **E + KK + M (2)** again:
We have **E** +16 + **M** is a two-digit number. The only unused squares of one or two digits are 9, 64 and 81, and of these only 9 and 64 give a value for 23ac which has two digits. **E** and **M** are therefore 9 and 64 in some order, and 23ac is therefore 89, necessarily entered in two cells.

10dn **DKS + K + KR (5)**
We have 4(**R** + 1) + 78400 = 8□□20 (not 8□209 because it's a multiple of 4), ie, 4**R** + 78404 = 8□□20. 4**R** must therefore end with 6. **R** therefore ends with 4 or 9. Since the value must lie in the range 81020 to 82520, we have 4**R** = 2616 to 4116, ie, **R** = 654 to 1029, ie, 26^2 to 32^2. The only unused squares in this range which end with 4 or 9 are 27^2 (= 729) and 32^2 (= 1024). R = 1024 gives 10dn = 82500, which doesn't fit, and so **R = 729**, making 10dn = 81320.

This is what the grid looks like now:

1	2 1 13	3 3 12	4 □□	5 17	2	6 13
7	8		9 13	10 8	11 7	12 2
		13		13		4
14		15	16	17 20	18 9	19 2
20	21	22	23 8	9	24 8	20
		25				9
26			27			

A =		N =	
B =	49	O =	
C =		P =	
D =	784	R =	729
E =		S =	25
F =		T =	
G =	2209	U =	36
H =		V =	
I =	121	W =	324
J =		X =	
K =	4	Y =	16
L =		Z =	
M =			

7ac **BR + BRU + U (7)**
This can now be evaluated as 1321713. 13217 therefore has to fit into three cells, and this can be done as 13 2 17 or 13 21 7.

4ac **2K + KR + W + Z (5)**
We have **Z** + 3248 = □□172, and so **Z** must end with 924, its square root therefore ending with 2 or 8. The value of 4ac must lie in the range 10172 to 25172, and so **Z** must be in the range 6924 to 21924. **Z = 13924** is the only perfect square which fits, making 4ac = 17172. 2ac is therefore to be entered as 13 12, since only two cells are available for it.

11dn **DX + UZ + 2Z (6)**
We have 784**X** + 36x13924 + 2x13924 = 7□98□□ or 7□□98□. Therefore, 784**X** + 529112 = 710980 to 799899, putting **X** in the range 232 to 345. **X** is therefore 256 or 289 (**W** is already 324). **X** = 289 gives a value of 755688, which doesn't fit, and so **X = 256**, giving a value for 11dn of 729816. We don't yet know whether the last two digits fit into one cell or two.

1dn **D + EZ + UU + Z (6)**
E is the only unknown value here, and we know that it is 9 or 64 from the clue for 23ac. **E** = 64 gives 907140, which doesn't fit, but **E = 9** gives a value of 141320, which does. This value can only be entered into three cells, because the 13 is already in the grid. Therefore, from 23ac, **M = 64**.

3dn **BP + SSU (6)**
This gives 49**P** + 25x25x36 = 1217□□ or 127□□□, and so we have 49**P** + 22500 = 121710 to 127999, putting **P** in the range 2025 to 2153. **P** is therefore 2025 or 2116. The latter value gives 3dn = 126184, which doesn't fit, and so **P = 2025** and 3dn = 121725. The second cell is therefore 17, not 7. We don't know yet how the last two digits are to be entered.

7ac **BR + BRU + U (7)** again:
This can now be entered as 13 2 17 13.

14dn **2P + Y + YYY (4)** again:
This can now be evaluated as 8162. The first cell therefore contains 8.

22ac **P + PT + R + RR (6)**
Only **T** is unknown. We have 2025**T** + 534195 = □89820, and so **T** must be odd. 2025**T** cannot exceed 989820 – 534195 = 455625, and so the maximum value of **T** is 225. The only lower unused odd squares are 81 and 169, giving 22ac = 698220 and 876420 respectively. These don't fit, and so **T = 225** and 22ac = 989820 — just the first 9 needs entering in the grid.

Over half the grid has now been filled:

(1) 14	(2) 13	(3) 12	(4) 17	(5) 17	2	(6) 13
(7) 13	(8) 2	17	(9) 13	(10) 8	(11) 7	(12) 2
20		(13) 2 25		13	2	4
(14) 8		(15)	(16)	(17) 20	(18) 9	(19) 2
(20)	(21)	(22) 9	(23) 8	9	(24) 8	20
		(25)			1 16	9
(26)			(27)			

A =		N =	
B =	49	O =	
C =		P =	2025
D =	784	R =	729
E =	9	S =	25
F =		T =	225
G =	2209	U =	36
H =		V =	
I =	121	W =	324
J =		X =	256
K =	4	Y =	16
L =		Z =	13924
M =	64		

4dn B + J + R + T (5)
This gives **J** + 1003 = 1713□, and so **J = 16129** and 4dn = 17132 — just the 2 needs to be entered.

20ac J (5)
Now that we know the value of **J**, we can enter 16 in cell 20 and 12 in cell 21, and as a result the last digit of 14dn (2) can be written in the grid.

13ac KY + V + Z (6)
Only **V** is unknown. We have **V** + 13988 = 221324 or 252132, and so **V** = 207336 or 238144. The first value is not a perfect square, and so **V = 238144** (= 488^2) and 13ac is 252132.

12dn H + III + T + V again:
Now that the value of **V** has been determined, we have **H** + 121^3 + 225 + 238144 = 242209□, giving **H = 412164** and 12dn = 2422094, entered as 2 4 2 20 9 4, to give this grid:

(1) 14	(2) 13	(3) 12	(4) 17	(5) 17	2	(6) 13
(7) 13	(8) 2	17	(9) 13	(10) 8	(11) 7	(12) 2
20		(13) 25	2	13	2	4
(14) 8		(15)	(16)	(17) 20	(18) 9	(19) 2
(20) 16	(21) 12	(22) 9	(23) 8	9	(24) 8	20
2		(25)			1 16	9
(26)			(27)			4

A =		N =	
B =	49	O =	
C =		P =	2025
D =	784	R =	729
E =	9	S =	25
F =		T =	225
G =	2209	U =	36
H =	412164	V =	238144
I =	121	W =	324
J =	16129	X =	256
K =	4	Y =	16
L =		Z =	13924
M =	64		

13dn J + OY + P (5)
This gives 16**O** + 18154 = 25□9□ or 25□□9. This puts **O** in the range 435 to 490, making **O** = 441 or 484. The former value gives 25210, which doesn't fit, making **O = 484** and 13dn = 25898. This can only be entered as 25 8 9 8.

14ac JK + KMM + O (5)

We can evaluate this as 16129x4 + 4x64x64 + 484 = 81384, to be entered as 8 13 8 4.

25ac F + FW + IIO (7)
We have 325**F** + 7086244 = 8□□□□16, 8□□□169 or 8□□□□19. The first case can be dismissed, because 325**F** must end in 0 or 5, making 25ac end in 4 or 9, not 6. 325**F** therefore ends with 25 or 75. However, it is not possible for it to end with 75 because any square multiple of 25 ends with 00 or 25, and so 325**F** + 7086244 = 8□□□169. The minimum possible value is 8101169 and the maximum is 8925169, putting **F** in the range 3123 to 5658, ie, **F** = 56^2 to 75^2. This means that 325**F** ends with 925, ie, 325**F** is of the form 1000k + 925. Dividing by 25, this gives 13**F** = 40k + 37. **F** must therefore end with 9.

The values to try are therefore 57^2, 63^2, 67^2 and 73^2, corresponding to 25ac = 8142169, 8376169, 8545169 and 8818169 respectively. The middle two of these can't be fitted according to the rules, and so 25ac is 8 14 2 16 9 (**F** = 57^2) or 8 8 18 16 9 (**F** = 73^2).

Note that by giving the matter a little thought, only four values of **F** needed to be tried – it wasn't necessary to use a program or spreadsheet.

Here is the almost-completed grid:

(1) 14	(2) 13	(3) 12	(4) 17	(5) 17	2	(6) 13
(7) 13	(8) 2	17	(9) 13	(10) 8	(11) 7	(12) 2
20		(13) 25	2	13	2	4
(14) 8	13	(15) 8	(16) 4	(17) 20	(18) 9	(19) 2
(20) 16	(21) 12	(22) 9	(23) 8	9	(24) 8	20
2		(25) 8	14 / 8	2 / 18	16	9
(26)			(27)			4

A =		**N =**	
B =	**49**	**O =**	**484**
C =		**P =**	**2025**
D =	**784**	**R =**	**729**
E =	**9**	**S =**	**25**
F =		**T =**	**225**
G =	**2209**	**U =**	**36**
H =	**412164**	**V =**	**238144**
I =	**121**	**W =**	**324**
J =	**16129**	**X =**	**256**
K =	**4**	**Y =**	**16**
L =		**Z =**	**13924**
M =	**64**		

16dn C + CU + N + 2O (5)
We have 37**C** + **N** + 968 = 48□□□ where the third cell contains 14 or 8. 37**C** + **N** is therefore in the range 48141 – 968 to 48825 – 968, ie, 47173 to 47857.

8dn C + II + 3N (5)
We have **C** + 14641 **+ 3N** = 2□□13. **C + 3N** is therefore in the range 21013 – 14641 to 22513 – 14641, ie, 6372 to 7872. Combining this result with the above, we have

37**C** + **N** = 47173 to 47857
and **C + 3N** = 6372 to 7872 (A)

So 111**C** + 3**N** = 3x47173 to 3x47857 (B)

Subtracting (A) from (B) and forcing the widest possible range, we have

110**C** = 3x47173 – 7872 to 3x47857 – 6372,

ie, 110**C** = 133647 to 137199.

This puts **C** in the range 1215 to 1247, and so **C = 1225**.

We can actually refine the range for 37**C** + **N**: in 16dn we have 37**C** + **N** + 968 = 48141 to 48149 *or* 48810 to 48825, ie, 37**C** + **N** = 47173 to 47181 *or* 47842 to 47857. Using the first range, we have **N** = 47173 – 37x1225 to 47181 – 37x1225 = 1848 to 1856, and the second range gives **N** = 47842 – 37x1225 to 47857 – 37x1225 = 2517 to 2532. The only perfect square within one of these ranges is **N = 1849**. 16dn is therefore 48 14 2 and 8dn is 2 14 13. Note once again that it was not necessary to plough through many possibilities (by using a program or spreadsheet) to find numbers which fit; all it required was a little thought.

15dn EK + EN + L + N (5)
L + 18526 = 898□□, giving **L** as roughly 71300, and so **L = 71289** and 15dn = 89815.

27ac 2C + EN + N + O (5)
This can now be evaluated. It turns out to be 21424, and since the first 2 is in the grid, it can be entered as 2 14 24 (4) or 2 14 2 4.

21dn 2A + N + S (5)
This reduces to 2**A** + 1874 = 12□□□, and so **A** must be in the range (12101 – 1874) / 2 to (12925 – 1874) / 2, ie, 5114 to 5525.

26ac A + 3C + EO (5)
We have **A** + 8031 = □□□15 or □□152, and so **A** ends with 84 or 121. **A = 5184** is the only square in the range 5114 to 5525 which fits. 26ac is therefore 13215, to be entered as 13 2 15, and 21dn is 12242. The grid is now complete, but there is an ambiguity in the bottom right of the grid:

(1) 14	(2) 13	(3) 12	(4) 17	(5) 17	2	(6) 13
(7) 13	(8) 2	17	(9) 13	(10) 8	(11) 7	(12) 2
20	14	(13) 25	2	13	2	4
(14) 8	13	(15) 8	(16) 4	(17) 20	(18) 9	(19) 2
(20) 16	(21) 12	(22) 9	(23) 8	9	(24) 8	20
2	24	(25) 8	14	2	16	9
(25) 13	2	15	(27) 2	14	2(4)	4

A =	**5184**	**N =**	**1849**
B =	**49**	**O =**	**484**
C =	**1225**	**P =**	**2025**
D =	**784**	**R =**	**729**
E =	**9**	**S =**	**25**
F =	**3249**	**T =**	**225**
G =	**2209**	**U =**	**36**
H =	**412164**	**V =**	**238144**
I =	**121**	**W =**	**324**
J =	**16129**	**X =**	**256**
K =	**4**	**Y =**	**16**
L =	**71289**	**Z =**	**13924**
M =	**64**		

Sorting the values into ascending order, we get

K	4
E	9
Y	16
S	25
U	36
B	49
M	64
I	121
T	225
X	256
W	324
O	484
R	729
D	784

C	1225
N	1849
P	2025
G	2209
F	3249
A	5184
Z	13924
J	16129
L	71289
V	238144
H	412164

The first 14 letters read KEYSUBMITXWORD, and we are told that this will indicate what solvers are to find and what they are to do after making changes to the grid contents. We are therefore to find KEY and then to SUBMIT XWORD, ie, a crossword. It is apparent, then that each number in the grid must be replaced by a letter, and we need a key to do it.

Many solvers' first thought will be to replace each number in the grid by the letter which has that position in the alphabet, but this does not produce anything relevant or interesting. However, replacing each number by the letter which has that position in the above list produces the grid below, and this is what each solver should submit:

1 D	2 R	3 O	4 P	5 P	E	6 R
7 R	8 E	P	9 R	10 I	11 M	12 E
A	D	13 H	E	R	E	S
14 I	R	15 I	16 S	17 A	18 T	19 E
20 N	21 O	22 T	23 I	T	24 I	A
E	V	25 I	D	E	N	T
26 R	E	C	27 E	D	E	S

This grid presents one of the few ways in which 14 words can be interlocked in this manner if foreign words, contractions and proper nouns are avoided.

Footnote

Each of the squares represented by the letters in the clues could be converted to abbreviations or words in *The Chambers Dictionary* by using the same key as above:

1	**K**	4	S
2	**E**	9	T
3	**Y**	16	N or KB
4	**S**	2 5	EU
5	**U**	3 6	Yb
6	**B**	4 9	St
7	**M**	6 4	BS
8	**I**	12 1	OK
9	**T**	2 25	eh
10	**X**	25 6	HB
11	**W**	3 2 4	yes
12	**O**	4 8 4	sis
13	**R**	7 2 9	met
14	**D**	7 8 4	mis

15	**C**	12 25	oh
16	**N**	1 8 4 9	kist
17	**P**	20 25	ah
18	**G**	2 20 9	eat
19	**F**	3 2 4 9	yest
20	**A**	5 18 4	ugs
21	**Z**	1 3 9 2 4	kytes
22	**J**	16 12 9	not
23	**L**	7 12 8 9	moit
24	**V**	23 8 14 4	lids
25	**H**	4 12 16 4	sons